rigourous) proofs. Descartes showed us that we could re-describe geometry using arithmetic. Hilbert sought to rid all geometric proofs of geometrical intuition, and replace them with formal, uninterpreted calculation. So we had a complete reversal, where first geometry set the higher standard of rigour and then algebra set the higher standard. All this was done in the name of increasing rigour.

In the paper, we will (1) list some of the different motivations for increasing rigour of a mathematical argument, then (2) we will search for a characterisation of rigour, by comparing it to the notion of gapless proof. (3) We then draw some conclusions from what we discovered in (1) and (2). In the conclusions we confirm that rigour is a relative term, and it varies with respect to quantity and quality (rigour in geometry is not obviously the same as rigour in arithmetic). To try to make an ordinal comparison between different (prima facie) qualities of rigour we have to reduce one type of proof to the other or express both proofs in a foundational theory and, philosophically, this is not a trivial matter. But *rigour* is not, for all that, an unconstrained term. There are completely definite things we can say, and some ordinal comparisons can be made, provided we understand the context for the comparison.

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A proof interpretation I of a theory T_1 in a theory T_2 is a function mapping a formula A of T_1 to a formula $A_I(x)$ of T_2 (with distinguished variables x) verifying $\mathsf{T}_1 \vdash A \Rightarrow \mathsf{T}_2 \vdash A_I(t)$ for some terms t extracted from a proof of A. Proof interpretations are used in:

- consistency results (e.g., if $\bot_I(x) = \bot$, then $\mathsf{T}_1 \vdash \bot \Rightarrow \mathsf{T}_2 \vdash \bot$, i.e., T_1 is consistent relatively to T_2);
- conservation results (e.g., if $\mathsf{T}_2 \vdash A_I(x) \to A$ for Π^0_2 formulas A, then $\mathsf{T}_1 \vdash A \Rightarrow \mathsf{T}_2 \vdash A$, i.e., T_1 is conservative over T_2 for Π^0_2 formulas);
- closure under rules (e.g., if $\mathsf{T}_1 = \mathsf{T}_2$, $(\exists y A(y))_I(x) = A_I(x)$ and $\mathsf{T}_1 \vdash A_I(x) \to A(x)$, then $\mathsf{T}_1 \vdash \exists y A(y) \Rightarrow \mathsf{T}_1 \vdash A(t)$ for some term t, i.e., T_1 has the existence property);
- extracting computational content from proofs (e.g., extracting t in the previous point).

The last two applications need $\mathsf{T}_1 \vdash A_I(x) \to A$ that can be achieved by:

- upgrading T_1 to the characterization theory CT that proves $\exists x A_I(x) \leftrightarrow A$;
- or hardwiring truth in I obtaining It verifying $\mathsf{T}_1 \vdash A_{It}(x) \to A$.

The first option doesn't work if:

- CT is classically inconsistent (e.g., bounded proof interpretations);
- or we want applications to theories weaker than CT.

So we turn to the second option and present a method to hardwire truth (in proof interpretations of Heyting arithmetic satisfying some mild conditions) by defining:

- a function c that replaces each subformula A of a formula by $A \wedge A_c$ where A_c is a "copy" of A;
- an "inverse" function c^{-1} that replaces A_c by A;
- $It = c^{-1} \circ I \circ c$.

As examples we hardwire truth in:

- modified realizability;
- Diller-Nahm functional interpretation;
- bounded modified realizability;
- bounded functional interpretation;
- slash.

This is based on joint work with Paulo Oliva [1].

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[1] JAIME GASPAR and PAULO OLIVA, *Proof interpretations with truth*, ***Mathematical Logic Quarterly***, forthcoming.

► ALEXANDER GAVRYUSHKIN, *New spectra of computable models*.
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The problem of description of spectra of computable models is nontrivial in the case where a theory is ω_1- and not ω-categorical. In the case of Ehrenfeucht theories, the computable models spectrum and the decidable models spectrum are both hard to describe. This paper adds several new spectra of computable models for the case of Ehrenfeucht theories to the list of known spectra.

A model is said to be *quasi-prime over a type p* if it is prime over a finite tuple of constants which realize the type p. A model $\mathfrak{M}$ is said to be *limit over a type p* if $\mathfrak{M} = \bigcup_{n\in\omega} \mathfrak{M}_n$, for some elementary chain $(\mathfrak{M}_n)_{n\in\omega}$ of quasi-prime models over p, and $\mathfrak{M} \not\cong \mathfrak{M}_p$. $\mathfrak{M}$ is said to be a *quasi-prime* (*limit*) *model* if it is quasi-prime (limit) over some type.

Every model of an Ehrenfeucht theory (a theory with finite, greater then 1, number of countable models) is either quasi-prime or limit.

A pair $\langle X, F\rangle$, where X is a pre-ordered finite set and $F\colon X \to \omega$, is said to be *e-parameters* of the theory T if (1) X is isomorphic to the set of all *quasi-prime* models of the theory T with the relation of elementary embeddability $\hookrightarrow$; (2) for all $x \in X$, the number of limit models over a quasi-prime model corresponding to the element x coincides with $F(x)$.

Spectrum of computable (*decidable*) *models* of an Ehrenfeucht theory T with e-parameters $\langle X, F\rangle$ is a pair $\langle Y, G\rangle$ where Y is a subset of X corresponding to the computable (decidable) quasi-prime models of the theory T, $G(x) \leqslant F(x)$ for all x, and the values of G correspond to the number of computable (decidable) limit models.

THEOREM 1. *Let* $n \geqslant 2$, $m \leqslant n-1$, *and the pairs* $\langle X, F\rangle$ *and* $\langle Y, G\rangle$ *are defined as follows*:

$$X = \{x_0 < x_1 \leqslant \cdots \leqslant x_n \mid x_n \leqslant \cdots \leqslant x_1\}, F(x_0) = 0, F(x_1) = \cdots = F(x_n) = 1;$$

$$Y = \{x_1 \leqslant \cdots \leqslant x_m \mid x_m \leqslant \cdots \leqslant x_1\}, G(x_0) = 0, G(x_1) = \cdots = G(x_n) = 1.$$

Then there is the theory T *satisfying the following conditions*:

1. $\langle X, F\rangle$ *are e-parameters of the theory* T;
2. T *has exactly* $n+2$ *non-isomorphic countable models*;
3. $\langle Y, G\rangle$ *is a spectrum of computable models of the theory* T.

Thus, for any m and n, there exist n $\hookrightarrow$-equivalent quasi-prime models among which exactly m models are computable. In the case of decidable models we have the following. If a quasi-prime model $\mathfrak{A}$ is $\hookrightarrow$-equivalent to a decidable one then $\mathfrak{A}$ is decidable.

THEOREM 2. *Let* $\langle X, F\rangle$ *be such that* $X = \{x_0 < x_1 < x_2\}$, $F(x_0) = F(x_1) = 0, F(x_2) = 3$, *and* $G\colon X \to \omega$ *is defined by* $G(x_0) = G(x_1) = 0$, $G(x_2) = 2$. *Then there is a theory* T *satisfying the following conditions*:

1. $\langle X, F\rangle$ *are e-parameters of the theory* T;
2. T *has exactly* 6 *non-isomorphic countable models*;
3. $\langle\{x_0\}, G\rangle$ *is a spectrum of computable models of the theory* T.

Thus, there are limit over the same type models one of which is computable and the other does not. In the case of decidable models, the question of existence of such models is known as the Morley Problem.